5-5 Solving Polynomial Equations

Factor completely. If the polynomial is not factorable, write *prime*.

1. $3ax + 2ay - az + 3bx + 2by - bz$

SOLUTION:

$$\begin{aligned} & 3ax+2ay-az+3bx+2by-bz \\ &=(3ax+2ay-az)+(3bx+2by-bz) && \text{Group to find a GCF.} \\ &=a(3a+2y-z)+b(3x+2y-z) && \text{Factor the GCF.} \\ &=(a+b)(3x+2y-z) && \text{Distributive Property} \end{aligned}$$

2. $2kx + 4mx - 2nx - 3ky - 6my + 3ny$

SOLUTION:

$$\begin{aligned} & 2kx+4mx-2nx-3ky-6my+3ny \\ &=(2kx+4mx-2nx)+(-3ky-6my+3ny) && \text{Group to find a GCF.} \\ &=2x(k+2m-n)-3y(2k+2m-n) && \text{Factor the GCF.} \\ &=(2x-3y)(k+2m-n) && \text{Distributive Property} \end{aligned}$$

3. $2x^3 + 5y^3$

SOLUTION:

Neither term is a perfect cube. The polynomial cannot be factored by cubic methods therefore it is a prime polynomial.

4. $16g^3 + 2h^3$

SOLUTION:

$$\begin{aligned} 16g^3+2h^3 &= 2\left(8g^3+h^3\right) && \text{Factor the GCF.} \\ &=2\left((2g)^3+h^3\right) && 8g^3=(2g)^3 \\ &=2(2g+h)\left(4g^2-2gh+h^2\right) && \text{Sum of two cubes} \end{aligned}$$

5. $12qw^3 - 12q^4$

SOLUTION:

$$\begin{aligned} 12qw^3-12q^4 &= 12q\left(w^3-q^3\right) && \text{Factor the GCF.} \\ &=12q(w-q)\left(w^2+wq+q^2\right) && \text{Difference of two cubes} \end{aligned}$$

6. $3w^2 + 5x^2 - 6y^2 + 2z^2 + 7a^2 - 9b^2$

SOLUTION:

There is no GCF and the polynomial cannot be factored by quadratic or cubic methods so it is prime

7. $a^6x^2 - b^6x^2$

SOLUTION:

$$a^6x^2 - b^6x^2$$

$$= x^2\left(a^6 - b^6\right) \quad \text{Factor the GCF.}$$

$$= x^2\left(\left(a^3\right)^2 - \left(b^3\right)^2\right) \quad a^6 = (a^3)^2,\ b^6 = (b^3)^2$$

$$= x^2\left(a^3 - b^3\right)\left(a^3 + b^3\right) \quad \text{Difference of two squares}$$

$$= x^2(a-b)\left(a^2 + ab + b^2\right)(a+b)\left(a^2 - ab + b^2\right) \quad \text{Sum and Difference of two cubes}$$

8. $x^3y2 - 8x^3\text{y} + 16x^3 + y^5 - 8y^4 + 16y^3$

SOLUTION:

$$x^3y^2 - 8x^3y + 16x^3 + y^5 - 8y^4 + 16y^3$$

$$= \left(x^3y^2 - 8x^3y + 16x^3\right) + \left(y^5 - 8y^4 + 16y^3\right) \quad \text{Group to find a GCF.}$$

$$= x^3\left(y^2 - 8y + 16\right) + y^3\left(y^2 - 8y + 16\right) \quad \text{Factor the GCF.}$$

$$= \left(x^3 + y^3\right)\left(y^2 - 8y + 16\right) \quad \text{Distributive Property}$$

$$= (x+y)\left(x^2 - xy + y^2\right)(y-4)^2 \quad \text{Simplify.}$$

9. $8c^3 - 125d^3$

SOLUTION:

$$8c^3 - 125d^3$$

$$= (2c)^3 - (5d)^3 \quad 8c^3 = (2c)^3,\ 125d^3 = (5d)^3$$

$$= (2c - 5d)\left(4c^2 + 10cd + 25d^2\right) \quad \text{Difference of two cubes}$$

10. $6bx + 12cx + 18dx - by - 2cy - 3dy$

SOLUTION:

$$6bx + 12cx + 18dx - by - 2cy - 3dy$$

$$= (6bx + 12cx + 18dx) + (-by - 2cy - 3dy) \quad \text{Group to find a GCF.}$$

$$= 6x(b + 2c + 3d) - y(b + 2c + 3d) \quad \text{Factor the GCF.}$$

$$= (6x - y)(b + 2c + 3d) \quad \text{Distributive Property}$$

5-5 Solving Polynomial Equations

Solve each equation.

11. $x^4 - 19x^2 + 48 = 0$

SOLUTION:

$$
\begin{aligned}
x^4 - 19x^2 + 48 &= 0 && \text{Original equation}\\
x^4 - (16+3)x^2 + 48 &= 0 && 19 = 16 + 3\\
x^4 - 16x^2 - 3x^2 + 48 &= 0 && \text{Distributive Property}\\
x^2\left(x^2 - 16\right) - 3\left(x^2 - 16\right) &= 0 && \text{Factor.}\\
\left(x^2 - 3\right)\left(x^2 - 16\right) &= 0 && \text{Distributive Property}
\end{aligned}
$$

$$
\begin{array}{lll}
x^2 - 3 = 0 & \text{or} & x^2 - 16 = 0\\
x^2 = 3 & \text{or} & x^2 = 16\\
x = \pm\sqrt{3} & \text{or} & x = \pm 4
\end{array}
$$

The solutions are ± 4 and $\pm\sqrt{3}$.

12. $x^3 - 64 = 0$

SOLUTION:

$$
\begin{aligned}
x^3 - 64 &= 0 && \text{Original equation}\\
x^3 - 4^3 &= 0 && 64 = 4^3\\
(x-4)\left(x^2 + 4x + 16\right) &= 0 && \text{Difference of two cubes}
\end{aligned}
$$

$$
\begin{array}{lll}
x - 4 = 0 & \text{or} & x^2 + 4x + 16 = 0
\end{array}
$$

Use the quadratic formula to factor $x^2 + 4x + 16$. This gives a solutions of $-2 \pm 2i\sqrt{3}$.
Therefore, the solution are $x = 4$, $-2 \pm 2i\sqrt{3}$.

5-5 Solving Polynomial Equations

13. $x^3 + 27 = 0$

SOLUTION:

$$x^3 + 27 = 0 \quad \text{Original equation}$$
$$x^3 + 3^3 = 0 \quad 27 = 3^3$$
$$(x+3)\left(x^2 - 3x + 9\right) = 0 \quad \text{Sum of two cubes}$$

$x + 3 = 0$ or $x^2 - 3x + 9 = 0$

One solution is $x = -3$.

Use the quadratic formula to factor $x^2 - 3x + 9$. Which is $\frac{3 \pm 3i\sqrt{3}}{2}$

Therefore, the solutions are $x = -3$ and $\frac{3 \pm 3i\sqrt{3}}{2}$.

14. $x^4 - 33x^2 + 200 = 0$

SOLUTION:

$x^4 - 33x^2 + 200 = 0$

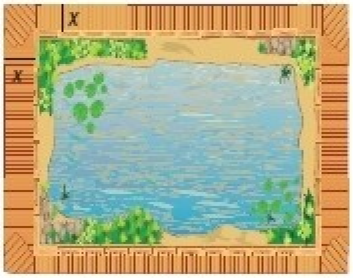
Let $y = x^2$.

$$y^2 - 33y + 200 = 0 \quad x^4 = y^2$$
$$(y-8)(y-25) = 0 \quad \text{Factor.}$$

$y - 8 = 0$	or	$y - 25 = 0$
$y = 8$	or	$y = 25$
$x^2 = 8$	or	$x^2 = 25$
$x = \pm 2\sqrt{2}$	or	$x = \pm 5$

Therefore, the solutions are 5, –5, $\pm 2\sqrt{2}$.

15. **CCSS PERSEVERANCE** A boardwalk that is x feet wide is built around a rectangular pond. The pond is 30 feet wide and 40 feet long. The combined area of the pond and the boardwalk is 2000 square feet. What is the width of the boardwalk?



SOLUTION:

The area of the pond is $30 \times 40 = 1200$ ft^2.
The area of the boardwalk is 2000 – 1200 = 800 ft^2.
The area of the board walk in terms of x is:

$$x(30)+x(30)+x(40)+x(40)+4x^2=4x^2+140x$$

Write an equation for the area of the board walk.

$$\begin{aligned}4x^2+140x&=800\\x^2+35x-200&=0\\(x+40)(x-5)&=0\end{aligned}$$

Therefore, x = –45 or x = 5
The value of x cannot be negative. Therefore, x = 5 ft.

Write each expression in quadratic form, if possible.

16. $4x^6 - 2x^3 + 8$

SOLUTION:

Let $y = (2x^3)$

$$\begin{aligned}4x^6-2x^3+8&=y^2-1y+8\\&=(2x^3)^2-1(2x^3)+8\end{aligned}$$

17. $25y^6 - 5y^2 + 20$

SOLUTION:

Since y^6 does not equal $(y^2)^2$ it is not possible to write this expression in quadratic form.

5-5 Solving Polynomial Equations

Solve each equation.

18. $x^4 - 6x^2 + 8 = 0$

SOLUTION:

Let $y = x^2$

$$\begin{aligned} x^4 - 6x^2 + 8 &= 0 && \text{Original equation} \\ y^2 - 6y + 8 &= 0 && \text{Substitute.} \\ (y-4)(y-2) &= 0 && \text{Factor.} \end{aligned}$$

$$\begin{array}{lcl} y - 4 = 0 & \text{or} & y - 2 = 0 \\ y = 4 & \text{or} & y = 2 \\ x^2 = 4 & \text{or} & x^2 = 2 \\ x = \pm 2 & \text{or} & x = \pm\sqrt{2} \end{array}$$

Therefore, the solutions are $2, -2, \sqrt{2}, -\sqrt{2}$.

19. $y^4 - 18y^2 + 72 = 0$

SOLUTION:

Let $x = y^2$

$$\begin{aligned} y^4 - 18y^2 + 72 &= 0 && \text{Original equation} \\ x^2 - 18x + 72 &= 0 && \text{Substitute.} \\ (x-12)(x-6) &= 0 && \text{Factor.} \end{aligned}$$

$$\begin{array}{lcl} x - 12 = 0 & \text{or} & x - 6 = 0 \\ x = 12 & \text{or} & x = 6 \\ y^2 = 12 & \text{or} & y^2 = 6 \\ y = \pm 2\sqrt{3} & \text{or} & y = \pm\sqrt{6} \end{array}$$

Therefore, the solutions are $\sqrt{6}, -\sqrt{6}, 2\sqrt{3}, -2\sqrt{3}$.

5-5 Solving Polynomial Equations

Factor completely. If the polynomial is not factorable, write *prime*.

20. $8c^3 - 27d^3$

SOLUTION:

$$\begin{aligned} & 8c^3 - 27d^3 & \\ &= (2c)^3 - (3d)^3 & 8c^3 = (2c)^3, 27d^3 = (3d)^3 \\ &= (2c - 3d)\left((2c)^2 + (2c)(3d) + (3d)^2\right) & \text{Difference of two cubes.} \\ &= (2c - 3d)\left(4c^2 + 6cd + 9d^2\right) & \text{Simplify.} \end{aligned}$$

21. $64x^4 + xy^3$

SOLUTION:

$$\begin{aligned} 64x^4 + xy^3 &= x\left(64x^3 + y^3\right) & \text{Factor the GCF.} \\ &= x\left((4x)^3 + y^3\right) & 64x^3 = (4x)^3 \\ &= x(4x + y)\left(16x^2 - 4xy + y^2\right) & \text{Sum of two cubes} \end{aligned}$$

22. $a^8 - a^2b^6$

SOLUTION:

$$\begin{aligned} & a^8 - a^2b^6 & \\ &= a^2\left(a^6 - b^6\right) & \text{Factor the GCF.} \\ &= a^2\left(\left(a^3\right)^2 - \left(b^3\right)^2\right) & a^6 = (a^3)^2, b^6 = (b^3)^2 \\ &= a^2\left(a^3 - b^3\right)\left(a^3 + b^3\right) & \text{Difference of two squares} \\ &= a^2(a - b)\left(a^2 + ab + b^2\right)(a + b)\left(a^2 - ab + b^2\right) & \text{Sum and difference of two cubes} \end{aligned}$$

23. $x^6y^3 + y^9$

SOLUTION:

$$\begin{aligned} x^6y^3 + y^9 &= y^3\left(x^6 + y^6\right) & \text{Factor the GCF.} \\ &= y^3\left(\left(x^2\right)^3 + \left(y^2\right)^3\right) & x^6 = (x^2)^3, y^6 = (y^2)^3 \\ &= y^3\left(x^2 + y^2\right)\left(x^4 - x^2y^2 + y^4\right) & \text{Simplify.} \end{aligned}$$

24. $18x^6 + 5y^6$

SOLUTION:

The polynomial cannot be factored using the sum of two cubes pattern or quadratic methods. Therefore it is prime.

25. $w^3 - 2y^3$

SOLUTION:

The polynomial cannot be factored by the difference of two cubes pattern or quadratic methods. Therefore it is prime.

26. $gx^2 - 3hx^2 - 6fy^2 - gy^2 + 6fx^2 + 3hy^2$

SOLUTION:

$$\begin{aligned}
&gx^2 - 3hx^2 - 6fy^2 - gy^2 + 6fx^2 + 3hy^2 \\
&= \left(gx^2 - 3hx^2 + 6fx^2\right) + \left(-6fy^2 - gy^2 + 3hy^2\right) && \text{Group to find a GCF.} \\
&= x^2(g - 3h + 6f) - y^2(6f + g - 3h) && \text{Factor the GCF.} \\
&= \left(x^2 - y^2\right)(6f + g - 3h) && \text{Distributive Property} \\
&= (x - y)(x + y)(6f + g - 3h) && \text{Factor.}
\end{aligned}$$

27. $12ax^2 - 20cy^2 - 18bx^2 - 10ay^2 + 15by^2 + 24cx^2$

SOLUTION:

$$\begin{aligned}
&12ax^2 - 20cy^2 - 18bx^2 - 10ay^2 + 15by^2 + 24cx^2 \\
&= \left(12ax^2 - 18bx^2 + 24cx^2\right) + \left(-20cy^2 - 10ay^2 + 15by^2\right) && \text{Group to find a GCF.} \\
&= 6x^2(2a - 3b + 4c) - 5y^2(2a - 3b + 4c) && \text{Factor the GCF.} \\
&= \left(6x^2 - 5y^2\right)(2a - 3b + 4c) && \text{Distributive Property}
\end{aligned}$$

28. $a^3x^2 - 16a^3x + 64a^3 - b^3x^2 + 16b^3x - 64b^3$

SOLUTION:

$$\begin{aligned}
&a^3x^2 - 16a^3x + 64a^3 - b^3x^2 + 16b^3x - 64b^3 \\
&= \left(a^3x^2 - 16a^3x + 64a^3\right) + \left(-b^3x^2 + 16b^3x - 64b^3\right) && \text{Group to find a GCF.} \\
&= a^3\left(x^2 - 16x + 64\right) - b^3\left(x^2 - 16x + 64\right) && \text{Factor the GCF.} \\
&= \left(a^3 - b^3\right)\left(x^2 - 16x + 64\right) && \text{Distributive Property} \\
&= (a - b)\left(a^2 + ab + b^2\right)(x - 8)^2 && \text{Factor.}
\end{aligned}$$

29. $8x^5 - 25y^3 + 80x^4 - x^2y^3 + 200x^3 - 10xy^3$

SOLUTION:

$$
\begin{aligned}
&8x^5 - 25y^3 + 80x^4 - x^2y^3 + 200x^3 - 10xy^3 \\
&= \left(8x^5 + 80x^4 + 200x^3\right) + \left(-25y^3 - x^2y^3 - 10xy^3\right) && \text{Group to find a GCF.} \\
&= 8x^3\left(x^2 + 10x + 25\right) - y^3\left(25 + x^2 - 10x\right) && \text{Factor the GCF.} \\
&= \left(8x^3 - y^3\right)\left(x^2 + 10x + 25\right) && \text{Distributive Property} \\
&= \left((2x)^3 - y^3\right)(x+5)(x+5) && \text{Factor.} \\
&= (2x - y)\left(4x^2 + 2xy + y^2\right)(x+5)^2 && \text{Factor.}
\end{aligned}
$$

Solve each equation.

30. $x^4 + x^2 - 90 = 0$

SOLUTION:

$x^4 + x^2 - 90 = 0$

Let $y = x^2$.

$$
\begin{aligned}
y^2 + y - 90 &= 0 && \text{Substitute.} \\
(y+10)(y-9) &= 0 && \text{Factor.}
\end{aligned}
$$

$$
\begin{array}{lll}
y + 10 = 0 & \text{or} & y - 9 = 0 \\
y = -10 & \text{or} & y = 9 \\
x^2 = -10 & \text{or} & x^2 = 9 \\
x = \pm i\sqrt{10} & \text{or} & x = \pm 3
\end{array}
$$

Therefore, the solutions are $3, -3, \pm i\sqrt{10}$.

5-5 Solving Polynomial Equations

31. $x^4 - 16x^2 - 720 = 0$

SOLUTION:

$x^4 - 16x^2 - 720 = 0$

Let $y = x^2$.

$$\begin{aligned} y^2 - 16y - 720 &= 0 \quad \text{Substitute.} \\ (y-36)(y+20) &= 0 \quad \text{Factor.} \end{aligned}$$

$$\begin{array}{lll} y - 36 = 0 & \text{or} & y + 20 = 0 \\ y = 36 & \text{or} & y = -20 \\ x^2 = 36 & \text{or} & x^2 = -20 \\ x = \pm 6 & \text{or} & x = \pm 2i\sqrt{5} \end{array}$$

Therefore, the solutions are $6, -6, \pm 2i\sqrt{5}$.

32. $x^4 - 7x^2 - 44 = 0$

SOLUTION:

$x^4 - 7x^2 - 44 = 0$

Let $y = x^2$.

$$\begin{aligned} y^2 - 7y - 44 &= 0 \quad \text{Substitute.} \\ (y-11)(y+4) &= 0 \quad \text{Factor.} \end{aligned}$$

$$\begin{array}{lll} y - 11 = 0 & \text{or} & y + 4 = 0 \\ y = 11 & \text{or} & y = -4 \\ x^2 = 11 & \text{or} & x^2 = -4 \\ x = \pm\sqrt{11} & \text{or} & x = \pm 2i \end{array}$$

Therefore, the solutions are $\pm\sqrt{11}, \pm 2i$.

5-5 Solving Polynomial Equations

33. $x^4 + 6x^2 - 91 = 0$

SOLUTION:

$x^4 + 6x^2 - 91 = 0$

Let $y = x^2$.

$$y^2 + 6y - 91 = 0 \quad \text{Substitute.}$$
$$(y + 13)(y - 7) = 0 \quad \text{Factor.}$$

$$\begin{array}{lcl} y - 7 = 0 & \text{or} & y + 13 = 0 \\ y = 7 & \text{or} & y = -13 \\ x^2 = 7 & \text{or} & x^2 = -13 \\ x = \pm\sqrt{7} & \text{or} & x = \pm i\sqrt{13} \end{array}$$

Therefore, the solutions are $\pm\sqrt{7}, \pm i\sqrt{13}$.

34. $x^3 + 216 = 0$

SOLUTION:

$$x^3 + 216 = 0 \quad \text{Original equation}$$
$$x^3 + 6^3 = 0 \quad 216 = 6^3$$
$$(x + 6)\left(x^2 - 6x + 36\right) = 0 \quad \text{Sum of two cubes}$$

$$\begin{array}{lcl} x + 6 = 0 & \text{or} & x^2 - 6x + 36 = 0 \\ x = -6 & \text{or} & x = \dfrac{6 \pm \sqrt{36 - 4(36)}}{2} \\ x = -6 & \text{or} & x = \dfrac{6 \pm \sqrt{-108}}{2} \\ x = -6 & \text{or} & x = \dfrac{6 \pm 6i\sqrt{3}}{2} \\ x = -6 & \text{or} & x = 3 \pm 3i\sqrt{3} \end{array}$$

Therefore, the solutions are $3 \pm 3i\sqrt{3}, -6$.

35. $64x^3 + 1 = 0$

SOLUTION:

$$64x^3+1=0 \quad \text{Original equation}$$
$$(4x)^3+1^3=0 \quad 64x^3=(4x)^3$$
$$(4x+1)\left(16x^2-4x+1\right)=0 \quad \text{Sum of two cubes}$$

$$4x+1=0 \quad \text{or} \quad 16x^2-4x+1=0$$
$$x=-\frac{1}{4} \quad \text{or} \quad x=\frac{4\pm\sqrt{16-4(16)}}{2(16)}$$
$$x=-\frac{1}{4} \quad \text{or} \quad x=\frac{4\pm\sqrt{-48}}{32}$$
$$x=-\frac{1}{4} \quad \text{or} \quad x=\frac{4\pm 4i\sqrt{3}}{32}$$
$$x=-\frac{1}{4} \quad \text{or} \quad x=\frac{1\pm i\sqrt{3}}{8}$$

Therefore, the solutions are $-\frac{1}{4}, \frac{1\pm i\sqrt{3}}{8}$.

Write each expression in quadratic form, if possible.

36. $x^4 + 12x^2 - 8$

SOLUTION:

Let $y = x^2$.

$$x^4+12x^2-8=y^2+12y-8$$
$$=\left(x^2\right)^2+12\left(x^2\right)-8$$

37. $-15x^4 + 18x^2 - 4$

SOLUTION:

Let $y = x^2$.

$$-15x^4+18x^2-4=-15y^2+18y-4$$

Substitute the value of y.

$$=-15\left(x^2\right)^2+18\left(x^2\right)-4$$

38. $8x^6 + 6x^3 + 7$

SOLUTION:

Let $y = 2x^3$.

$$8x^6 + 6x^3 + 7 = y^2 + 3y + 7$$
$$= 2\left(2x^3\right)^2 + 3\left(2x^3\right) + 7$$

39. $5x^6 - 2x^2 + 8$

SOLUTION:

This cannot be written in quadratic form since $x^6 \neq \left(x^2\right)^2$.

40. $9x^8 - 21x^4 + 12$

SOLUTION:

Let $y = 3x^4$.

$$9x^8 - 21x^4 + 12 = y^2 - 7y + 12$$
$$= \left(3x^4\right)^2 - 7\left(3x^4\right) + 12$$

41. $16x^{10} + 2x^5 + 6$

SOLUTION:

Let $y = 2x^5$.

$$16x^{10} + 2x^5 + 6 = 4y^2 + 1y + 6$$
$$= 4\left(2x^5\right)^2 + 1\left(2x^5\right) + 6$$

Solve each equation.

42. $x^4 + 6x^2 + 5 = 0$

SOLUTION:

$x^4 + 6x^2 + 5 = 0$

Let $y = x^2$.

$$y^2 + 6y + 5 = 0 \quad \text{Substitute.}$$
$$(y+5)(y+1) = 0 \quad \text{Factor.}$$

$y + 5 = 0$	or	$y + 1 = 0$
$y = -5$	or	$y = -1$
$x^2 = -5$	or	$x^2 = -1$
$x = \pm i\sqrt{5}$	or	$x^2 = \pm i$

The solutions are $\pm i\sqrt{5}, \pm i$.

43. $x^4 - 3x^2 - 10 = 0$

SOLUTION:

$x^4 - 3x^2 - 10 = 0$

Let $y = x^2$.

$$y^2 - 3y - 10 = 0 \quad \text{Substitute.}$$
$$(y-5)(y+2) = 0 \quad \text{Factor.}$$

$y - 5 = 0$	or	$y + 2 = 0$
$y = 5$	or	$y = -2$
$x^2 = 5$	or	$x^2 = -2$
$x = \pm\sqrt{5}$	or	$x = \pm i\sqrt{2}$

The solutions are $\pm\sqrt{5}, \pm i\sqrt{2}$.

44. $4x^4 - 14x^2 + 12 = 0$

SOLUTION:

$4x^4 - 14x^2 + 12 = 0$

Let $y = 2x^2$.

$$\begin{aligned} y^2 - 7y + 12 &= 0 && \text{Substitute.} \\ (y-4)(y-3) &= 0 && \text{Factor.} \end{aligned}$$

$$\begin{array}{lcl} y - 4 = 0 & \text{or} & y - 3 = 0 \\ y = 4 & \text{or} & y = 3 \\ 2x^2 = 4 & \text{or} & 2x^2 = 3 \\ x^2 = 2 & \text{or} & x^2 = \frac{3}{2} \\ x = \pm\sqrt{2} & \text{or} & x = \pm\sqrt{\frac{3}{2}} = \pm\frac{\sqrt{6}}{2} \end{array}$$

The solutions are $\pm\sqrt{2}, \pm\frac{\sqrt{6}}{2}$.

45. $9x^4 - 27x^2 + 20 = 0$

SOLUTION:

$9x^4 - 27x^2 + 20 = 0$

Let $y = 3x^2$.

$$\begin{aligned} y^2 - 9y + 20 &= 0 && \text{Substitute.} \\ (y-5)(y-4) &= 0 && \text{Factor.} \end{aligned}$$

$$\begin{array}{lcl} y - 4 = 0 & \text{or} & y - 5 = 0 \\ y = 4 & \text{or} & y = 5 \\ 3x^2 = 4 & \text{or} & 3x^2 = 5 \\ x^2 = \frac{4}{3} & \text{or} & x^2 = \frac{5}{3} \\ x = \pm\sqrt{\frac{4}{3}} & \text{or} & x = \pm\sqrt{\frac{5}{3}} \\ x = \pm\frac{2\sqrt{3}}{3} & \text{or} & x = \pm\frac{\sqrt{15}}{3} \end{array}$$

The solutions are $\pm\frac{2\sqrt{3}}{3}, \pm\frac{\sqrt{15}}{3}$.

46. $4x^4 - 5x^2 - 6 = 0$

SOLUTION:

$4x^4 - 5x^2 - 6 = 0$

Let $y = x^2$.

$$\begin{aligned} 4y^2 - 5y - 6 &= 0 && \text{Substitute.} \\ (4y+3)(y-2) &= 0 && \text{Factor.} \end{aligned}$$

$$\begin{array}{lcl} 4y+3=0 & \text{or} & y-2=0 \\ y=-\dfrac{3}{4} & \text{or} & y=2 \\ x^2=-\dfrac{3}{4} & \text{or} & x^2=2 \\ x=\pm i\dfrac{\sqrt{3}}{2} & \text{or} & x=\pm\sqrt{2} \end{array}$$

The solutions are $\pm i\dfrac{\sqrt{3}}{2}, \pm\sqrt{2}$.

47. $24x^4 + 14x^2 - 3 = 0$

SOLUTION:

$24x^4 + 14x^2 - 3 = 0$

Let $y = 2x^2$.

$$6y^2 + 7y - 3 = 0 \quad \text{Substitute.}$$
$$(3y - 1)(2y + 3) = 0 \quad \text{Factor.}$$

$$3y - 1 = 0 \quad \text{or} \quad 2y + 3 = 0$$
$$y = \frac{1}{3} \quad \text{or} \quad y = -\frac{3}{2}$$
$$2x^2 = \frac{1}{3} \text{ or } \quad 2x^2 = -\frac{3}{2}$$
$$x^2 = \frac{1}{6} \text{ or } \quad x^2 = -\frac{3}{4}$$
$$x = \pm\frac{1}{\sqrt{6}} \quad \text{or} \quad x = \pm i\frac{\sqrt{3}}{2}$$
$$x = \pm\frac{\sqrt{6}}{6} \quad \text{or} \quad x = \pm i\frac{\sqrt{3}}{2}$$

The solutions are $\pm i\frac{\sqrt{3}}{2}, \pm\frac{\sqrt{6}}{6}$.

48. **ZOOLOGY** A species of animal is introduced to a small island. Suppose the population of the species is represented by $P(t) = -t^4 + 9t^2 + 400$, where t is the time in years. Determine when the population becomes zero.

SOLUTION:

Substitute 0 for $P(t)$ and solve for t.

$$-t^4 + 9t^2 + 400 = 0$$

Let $x = t^2$.

$$-x^2 + 9x + 400 = 0$$
$$(-x - 16)(x - 25) = 0$$

Therefore, $x = 25$ or -16.

-16 is irrelevant because t^2 cannot be negative.

So, $t^2 = 25$
$t = \pm 5$

-5 is irrelevant because t cannot be negative.
Therefore, the population becomes zero in 5 years.

Factor completely. If the polynomial is not factorable, write prime.

49. $x^4 - 625$

SOLUTION:

$$\begin{aligned} x^4 - 625 &= \left(x^2\right)^2 - \left(5^2\right)^2 && x^4 = (x^2)^2 \\ &= \left(x^2 - 5^2\right)\left(x^2 + 5^2\right) && \text{Difference of two squares} \\ &= (x-5)(x+5)\left(x^2 + 5^2\right) && \text{Factor.} \\ &= (x-5)(x+5)\left(x^2 + 25\right) \end{aligned}$$

5-5 Solving Polynomial Equations

50. $x^6 - 64$

SOLUTION:

$$x^6 - 64$$

$$= \left(x^3\right)^2 - \left(2^3\right)^2 \quad \text{Difference of two squares}$$

$$= \left(x^3 - 2^3\right)\left(x^3 + 2^3\right) \quad \text{Factor.}$$

$$= (x-2)\left(x^2 + 2x + 4\right)\left(x^3 + 2^3\right) \quad \text{Difference of two cubes}$$

$$= (x-2)\left(x^2 + 2x + 4\right)(x+2)\left(x^2 - 2x + 4\right) \quad \text{Sum of two cubes}$$

51. $x^5 - 16x$

SOLUTION:

$$\begin{aligned} x^5 - 16x &= x\left(x^4 - 16\right) \\ &= x\left(\left(x^2\right)^2 - \left(2^2\right)^2\right) \\ &= x\left(x^2 - 2^2\right)\left(x^2 + 2^2\right) \\ &= x(x+2)(x-2)\left(x^2 + 4\right) \end{aligned}$$

52. $8x^5y^2 - 27x^2y^5$

SOLUTION:

$$\begin{aligned} 8x^5y^2 - 27x^2y^5 &= x^2y^2\left(8x^3 - 27y^3\right) \\ &= x^2y^2\left((2x)^3 - (3y)^3\right) \\ &= x^2y^2(2x - 3y)\left(4x^2 + 6xy + 9y^2\right) \end{aligned}$$

53. $15ax - 10bx + 5cx + 12ay - 8by + 4cy + 15az - 10bz + 5cz$

SOLUTION:

$$15ax - 10bx + 5cx + 12ay - 8by + 4cy + 15az - 10bz + 5cz$$

$$= \left(15ax - 10bx + 5cx\right) + \left(12ay - 8by + 4cy\right) + \left(15az - 10bz + 5cz\right)$$

$$= 5x\left(3a - 2b + c\right) + 4y\left(3a - 2b + c\right) + 5z\left(3a - 2b + c\right)$$

$$= \left(5x + 4y + 5z\right)\left(3a - 2b + c\right)$$

54. $6a^2x^2 - 24b^2x^2 + 18c^2x^2 - 5a^2y^3 + 20b^2y^3 - 15c^2y^3 + 2a^2z^2 - 8b^2z^2 + 6c^2z^2$

SOLUTION:

$$\begin{aligned}&6a^2x^2 - 24b^2x^2 + 18c^2x^2 - 5a^2y^3 + 20b^2y^3 - 15c^2y^3 + 2a^2z^2 - 8b^2z^2 + 6c^2z^2\\&= \left(6a^2x^2 - 24b^2x^2 + 18c^2x^2\right) + \left(-5a^2y^3 + 20b^2y^3 - 15c^2y^3\right) + \left(2a^2z^2 - 8b^2z^2 + 6c^2z^2\right)\\&= 6x^2\left(a^2 - 4b^2 + 3c^2\right) - 5y^3\left(a^2 - 4b^2 + 3c^2\right) + 2z^2\left(a^2 - 4b^2 + 3c^2\right)\\&= \left(6x^2 - 5y^3 + 2z^2\right)\left(a^2 - 4b^2 + 3c^2\right)\end{aligned}$$

55. $6x^5 - 11x^4 - 10x^3 - 54x^3 + 99x^2 + 90x$

SOLUTION:

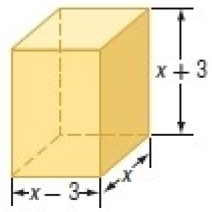
$$\begin{aligned}&6x^5 - 11x^4 - 10x^3 - 54x^3 + 99x^2 + 90x\\&= 6x^5 - 11x^4 - 64x^3 + 99x^2 + 90x\\&= x\left(6x^4 - 11x^3 - 64x^2 + 99x + 90\right)\\&= x(x+3)\left(6x^3 - 29x^2 + 23x + 30\right)\\&= x(x+3)(x-3)\left(6x^2 - 11x - 10\right)\\&= x(x+3)(x-3)(2x-5)(3x+2)\end{aligned}$$

56. $20x^6 - 7x^5 - 6x^4 - 500x^4 + 175x^3 + 150x^2$

SOLUTION:

$$\begin{aligned}&20x^6 - 7x^5 - 6x^4 - 500x^4 + 175x^3 + 150x^2\\&= 20x^6 - 7x^5 - 506x^4 + 175x^3 + 150x^2\\&= x^2\left(20x^4 - 7x^3 - 506x^2 + 175x + 150\right)\\&= x^2(x-5)\left(20x^3 + 93x^2 - 41x - 30\right)\\&= x^2(x-5)(x+5)\left(20x^2 - 7x + 6\right)\\&= x^2(x-5)(x+5)(4x-3)(5x+2)\end{aligned}$$

57. **GEOMETRY** The volume of the figure at the right is 440 cubic centimeters. Find the value of x and the length, height, and width.



SOLUTION:

The volume of the figure in terms of x is $(x+3)(x-3)x$.

Therefore, $x(x-3)(x+3)=440$.
Solve for x.

$$\begin{aligned} x(x^2-9)&=440 \\ x^3-9x-440&=0 \\ (x-8)(x^2+8x+55)&=0 \end{aligned}$$

The expression $x^2 + 8x + 55$ is a prime.
Therefore, $x = 8$.
The length is $x = 8$.
The height is $x + 3 = 11$.
The width is $x – 3 = 5$.

Solve each equation.

58. $8x^4 + 10x^2 - 3 = 0$

SOLUTION:

$$8x^4 + 10x^2 - 3 = 0$$

Let $u = 2x^2$.

$$2u^2 + 5u - 3 = 0$$
$$(u+3)(2u-1) = 0$$

$$\begin{array}{lll} u+3=0 & \text{or} & 2u-1=0 \\ u=-3 & \text{or} & u=\frac{1}{2} \\ 2x^2=-3 & \text{or} & 2x^2=\frac{1}{2} \\ x^2=-\frac{3}{2} & \text{or} & x^2=\frac{1}{4} \\ x=\pm i\frac{\sqrt{3}}{\sqrt{2}} & \text{or} & x=\pm\frac{1}{2} \\ x=\pm i\frac{\sqrt{6}}{2} & \text{or} & x=\pm\frac{1}{2} \end{array}$$

The solutions are $\pm\frac{1}{2}, \pm i\frac{\sqrt{6}}{2}$.

59. $6x^4 - 5x^2 - 4 = 0$

SOLUTION:

$6x^4 - 5x^2 - 4 = 0$

Let $u = x^2$.

$$\begin{aligned} 6u^2 - 5u - 4 &= 0 \\ (3u-4)(2u+1) &= 0 \end{aligned}$$

$$\begin{array}{lcl} 3u - 4 = 0 & \text{or} & 2u + 1 = 0 \\ u = \frac{4}{3} & \text{or} & u = -\frac{1}{2} \\ x^2 = \frac{4}{3} & \text{or} & x^2 = -\frac{1}{2} \\ x = \pm\frac{2}{\sqrt{3}} & \text{or} & x = \pm i\frac{1}{\sqrt{2}} \\ x = \pm\frac{2\sqrt{3}}{3} & \text{or} & x = \pm i\frac{\sqrt{2}}{2} \end{array}$$

The solutions are $\pm\frac{2\sqrt{3}}{3}, \pm i\frac{\sqrt{2}}{2}$.

60. $20x^4 - 53x^2 + 18 = 0$

SOLUTION:

$20x^4 - 53x^2 + 18 = 0$

Let $u = x^2$.

$$20u^2 - 53u + 18 = 0$$
$$(4u - 9)(5u - 2) = 0$$

$$\begin{array}{lll} 4u - 9 = 0 & \text{or} & 5u - 2 = 0 \\ u = \frac{9}{4} & \text{or} & u = \frac{2}{5} \\ x^2 = \frac{9}{4} & \text{or} & x^2 = \frac{2}{5} \\ x = \pm\frac{3}{2} & \text{or} & x = \pm\frac{\sqrt{2}}{\sqrt{5}} \\ x = \pm\frac{3}{2} & \text{or} & x = \pm\frac{\sqrt{10}}{5} \end{array}$$

The solutions are $\pm\frac{3}{2}, \pm\frac{\sqrt{10}}{5}$.

61. $18x^4 + 43x^2 - 5 = 0$

SOLUTION:

$$18x^4 + 43x^2 - 5 = 0$$

Let $u = x^2$.

$$18u^2 + 43u^2 - 5 = 0$$
$$(9u - 1)(2u + 5) = 0$$

$$\begin{array}{lcl} 9u - 1 = 0 & \text{or} & 2u + 5 = 0 \\ u = \frac{1}{9} & \text{or} & u = -\frac{5}{2} \\ x^2 = \frac{1}{9} & \text{or} & x^2 = -\frac{5}{2} \\ x = \pm\frac{1}{3} & \text{or} & x = \pm i\frac{\sqrt{5}}{\sqrt{2}} \\ x = \pm\frac{1}{3} & \text{or} & x = \pm i\frac{\sqrt{10}}{2} \end{array}$$

The solutions are $\pm\frac{1}{3}, \pm i\frac{\sqrt{10}}{2}$.

62. $8x^4 - 18x^2 + 4 = 0$

SOLUTION:

$$8x^4 - 18x^2 + 4 = 0$$

Let $u = x^2$.

$$8u^2 - 18u^2 + 4 = 0$$
$$(u - 2)(8u - 2) = 0$$

$$\begin{array}{lcl} u - 2 = 0 & \text{or} & 8u - 2 = 0 \\ u = 2 & \text{or} & u = \frac{1}{4} \\ x^2 = 2 & \text{or} & x^2 = \frac{1}{4} \\ x = \pm\sqrt{2} & \text{or} & x = \pm\frac{1}{2} \end{array}$$

The solutions are $\pm\sqrt{2}, \pm\frac{1}{2}$.

63. $3x^4 - 22x^2 - 45 = 0$

SOLUTION:

$$3x^4 - 22x^2 - 45 = 0$$

Let $u = x^2$.

$$\begin{aligned} 3u^2 - 22u^2 - 45 &= 0 \\ (3u+5)(u-9) &= 0 \end{aligned}$$

$$\begin{array}{lcl} u - 9 = 0 & \text{or} & 3u + 5 = 0 \\ u = 9 & \text{or} & u = -\dfrac{5}{3} \\ x^2 = 9 & \text{or} & x^2 = -\dfrac{5}{3} \\ x = \pm 3 & \text{or} & x = \pm i\dfrac{\sqrt{5}}{\sqrt{3}} \\ x = \pm 3 & \text{or} & x = \pm i\dfrac{\sqrt{15}}{3} \end{array}$$

The solutions are $\pm 3, \pm i\dfrac{\sqrt{15}}{3}$.

5-5 Solving Polynomial Equations

64. $x^6 + 7x^3 - 8 = 0$

SOLUTION:

$x^6 + 7x^3 - 8 = 0$

Let $u = x^3$.

$$\begin{aligned} u^2 + 7u - 8 &= 0 \\ (u+8)(u-1) &= 0 \end{aligned}$$

$$\begin{array}{lcl} u - 1 = 0 & \text{or} & u + 8 = 0 \\ x^3 - 1 = 0 & \text{or} & x^3 + 8 = 0 \end{array}$$

Solve each equation for x.

$$\begin{aligned} x^3 - 1 &= 0 \\ (x-1)(x^2 + x + 1) &= 0 \end{aligned}$$

$$\begin{aligned} x - 1 = 0 \quad \text{or} \quad & x^2 - x + 1 = 0 \\ x = 1 \quad \text{or} \quad & x = \frac{-(1) \pm \sqrt{(1)^2 - 4(1)(1)}}{2(1)} \\ & = \frac{-1 \pm \sqrt{-3}}{2} \\ & = \frac{-1 \pm i\sqrt{3}}{2} \end{aligned}$$

$$\begin{aligned} x^3 + 8 &= 0 \\ (x+2)(x^2 - 2x + 4) &= 0 \end{aligned}$$

$$\begin{aligned} x + 2 = 0 \quad \text{or} \quad & x^2 - 2x + 4 = 0 \\ x = -2 \quad \text{or} \quad & x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(4)}}{2(1)} \\ & = \frac{2 \pm \sqrt{-12}}{2} \\ & = \frac{2 \pm 2i\sqrt{3}}{2} \\ & = 1 \pm i\sqrt{3} \end{aligned}$$

The solutions are 1, –2, $\frac{1 \pm \sqrt{-3}}{2}$, and $1 \pm i\sqrt{3}$

65. $x^6 - 26x^3 - 27 = 0$

SOLUTION:

$$x^6 - 26x^3 - 27 = 0$$

Let $u = x^3$.

$$u^2 - 26u - 27 = 0$$
$$(u-27)(u+1) = 0$$

$$u + 1 = 0 \quad \text{or} \quad u - 27 = 0$$
$$x^3 + 1 = 0 \quad \text{or} \quad x^3 - 27 = 0$$

Solve each equation for x.

$$x^3 + 1 = 0$$
$$(x+1)(x^2 + x + 1) = 0$$

$$x + 1 = 0 \quad \text{or} \quad x^2 - x + 1 = 0$$
$$x = -1 \quad \text{or} \quad x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(1)}}{2(1)}$$
$$= \frac{1 \pm \sqrt{-3}}{2}$$
$$= \frac{1 \pm \boldsymbol{i}\sqrt{3}}{2}$$

$$x^3 - 27 = 0$$
$$(x-3)(x^2 + 3x + 9) = 0$$

$$x - 3 = 0 \quad \text{or} \quad x^2 + 3x + 9 = 0$$
$$x = 3 \quad \text{or} \quad x = \frac{-(3) \pm \sqrt{(3)^2 - 4(1)(9)}}{2(1)}$$
$$= \frac{-3 \pm \sqrt{-27}}{2}$$
$$= \frac{-3 \pm 3\boldsymbol{i}\sqrt{3}}{2}$$

The solutions are –1, 3, $\frac{-3 \pm 3\boldsymbol{i}\sqrt{3}}{2}$, and $\frac{1 \pm \boldsymbol{i}\sqrt{3}}{2}$.

66. $8x^6 + 999x^3 = 125$

SOLUTION:

$8x^6 + 999x^3 = 125$

Let $u = x^3$.

$$8u^2 + 999u - 125 = 0$$
$$(8u - 1)(u + 125) = 0$$

$$8u - 1 = 0 \quad \text{or} \quad u + 125 = 0$$
$$x^3 - 1 = 0 \quad \text{or} \quad x^3 + 125 = 0$$

Solve each equation for x.

$$x^3 - 1 = 0$$
$$(x - 1)(x^2 + x + 1) = 0$$

$$x - 1 = 0 \quad \text{or} \quad x^2 - x + 1 = 0$$
$$x = 1 \quad \text{or} \quad x = \frac{-(1) \pm \sqrt{(1)^2 - 4(1)(1)}}{2(1)}$$
$$= \frac{-1 \pm \sqrt{-3}}{2}$$
$$= \frac{-1 \pm i\sqrt{3}}{2}$$

$$x^3 + 125 = 0$$
$$(x + 5)(x^2 - 5x + 25) = 0$$

$$x + 5 = 0 \quad \text{or} \quad x^2 - 5x + 25 = 0$$
$$x = -5 \quad \text{or} \quad x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(1)(25)}}{2(1)}$$
$$= \frac{5 \pm \sqrt{-75}}{2}$$
$$= \frac{5 \pm 5i\sqrt{3}}{2}$$

The solutions are –5, $\frac{1}{2}$, $\frac{-1 \pm i\sqrt{3}}{2}$, and $\frac{5 \pm 5i\sqrt{3}}{2}$.

67. $4x^4 - 4x^2 - x^2 + 1 = 0$

SOLUTION:

$$4x^4 - 4x^2 - x^2 + 1 = 0$$
$$4x^4 - 5x^2 + 1 = 0$$

Let $u = x^2$.

$$4u^2 - 5u + 1 = 0$$
$$(4u - 1)(u - 1) = 0$$

By Zero Product Property:

$$\begin{array}{lll} 4u - 1 = 0 & \text{or} & u - 1 = 0 \\ u = \frac{1}{4} & \text{or} & u = 1 \\ x^2 = \frac{1}{4} & \text{or} & x^2 = 1 \\ x = \pm\frac{1}{2} & \text{or} & x = \pm 1 \end{array}$$

The solutions are $\pm\frac{1}{2}, \pm 1$.

68. $x^6 - 9x^4 - x^2 + 9 = 0$

SOLUTION:

$$x^6 - 9x^4 - x^2 + 9 = 0$$

Let $u = x^2$.

$$u^3 - 9u^2 - u + 9 = 0$$
$$(u - 1)(u^2 - 8u - 9) = 0$$
$$(u - 1)(u + 1)(u - 9) = 0$$

By Zero Product Property:

$$\begin{array}{lllll} u - 1 = 0 & \text{or} & u + 1 = 0 & \text{or} & u - 9 = 0 \\ u = 1 & \text{or} & u = -1 & \text{or} & u = 9 \\ x^2 = 1 & \text{or} & x^2 = -1 & \text{or} & x^2 = 9 \\ x = \pm 1 & \text{or} & x = \pm i & \text{or} & x = \pm 3 \end{array}$$

The solutions are $\pm i, \pm 1, \pm 3$.

69. $x^4 + 8x^2 + 15 = 0$

SOLUTION:

$x^4 + 8x^2 + 15 = 0$

Let $u = x^2$.

$$u^2 + 8u + 15 = 0$$
$$(u + 5)(u + 3) = 0$$

By Zero Product Property:

$u + 5 = 0$	or	$u + 3 = 0$
$u = -5$	or	$u = -3$
$x^2 = -5$	or	$x^2 = -3$
$x = \pm i\sqrt{5}$	or	$x = \pm i\sqrt{3}$

The solutions are $\pm i\sqrt{3}, \pm i\sqrt{5}$.

70. **CCSS SENSE-MAKING** A rectangular prism with dimensions $x - 2$, $x - 4$, and $x - 6$ has a volume equal to $40x$ cubic units.

a. Write out a polynomial equation using the formula for volume.

b. Use factoring to solve for x.

c. Are any values for x unreasonable? Explain.

d. What are the dimensions of the prism?

SOLUTION:

a. The volume of the prism is $(x-2)(x-4)(x-6)$.

$$(x-2)(x-4)(x-6) = x^3 - 12x^2 + 44x - 48$$

Therefore, $x^3 - 12x^2 + 44x - 48 = 40x$

b. Solve for x.

$$\begin{aligned} x^3 - 12x^2 + 44x - 48 &= 40x \\ x^3 - 12x^2 + 4x - 48 &= 0 \\ (x-12)(x^2+4) &= 0 \end{aligned}$$

By Zero Product Property:

$$\begin{array}{lcl} x - 12 = 0 & \text{or} & x^2 + 4 = 0 \\ x = 12 & \text{or} & x^2 = -4 \\ x = 12 & \text{or} & x = \pm 2i \end{array}$$

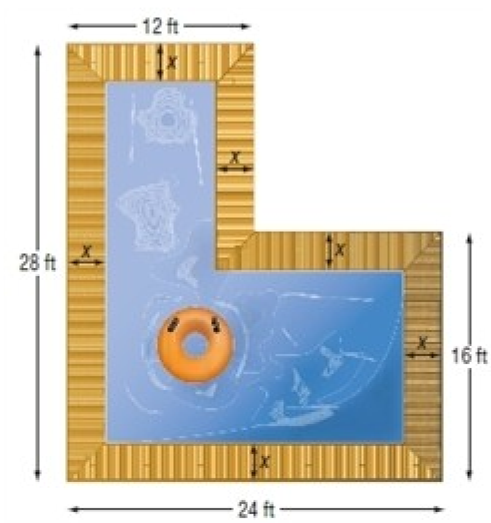
The solutions are $\pm 2i$ and 12.

c. Sample answer: ±2i because they are imaginary numbers.

d. $x - 2 = 12 - 2 = 10$
$x - 4 = 12 - 4 = 8$
$x - 6 = 12 - 6 = 6$

The dimensions are 6, 8 and 10 units.

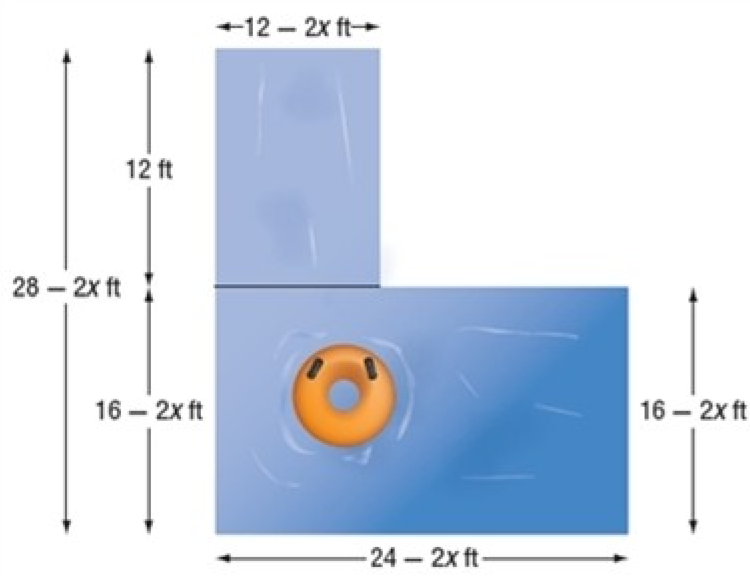
71. **POOL DESIGN** Andrea wants to build a pool following the diagram at the right. The pool will be surrounded by a sidewalk of a constant width.



a. If the total area of the pool itself is to be 336 ft^2, what is x?
b. If the value of x were doubled, what would be the new area of the pool?
c. If the value of x were halved, what would be the new area of the pool?

SOLUTION:

a. Let x be the width of the sidewalk.
The dimensions of pool is $16 - 2x$, $24 - 2x$, $28 - 2x$ and $12 - 2x$.



The area of the pool in terms of x is $(16-2x)(24-2x)+12(12-2x)$.
Equate the area of the pool with the expression and solve for x.

$$\begin{aligned}(16-2x)(24-2x)+12(12-2x)&=336\\384+4x^2-80x+144-24x&=336\\4x^2-104x+192&=0\\x^2-26x+48&=0\\(x-24)(x-2)&=0\end{aligned}$$

Therefore, $x = 24$ or $x = 2$.
The value of x cannot be 24. So $x = 2$ ft.

b. Substitute 4 for x in the area of pool and simplify.

$$\begin{aligned}&(16-2x)(24-2x)+12(12-2x)\\&=(16-2(4))(24-2(4))+12(12-2(4))\\&=8(16)+12(4)\\&=176\end{aligned}$$

The new area of the pool is 176 ft^2.

c. Substitute 1 for x in the area of pool and simplify.

$$\begin{aligned}&(16-2x)(24-2x)+12(12-2x)\\&=(16-2(1))(24-2(1))+12(12-2(1))\\&=14(22)+12(10)\\&=428\end{aligned}$$

The new area of the pool is 428 ft^2.

72. **BIOLOGY** During an experiment, the number of cells of a virus can be modeled by $P(t) = -0.012t^3 - 0.24t^2 + 6.3t + 8000$, where t is the time in hours and P is the number of cells. Jack wants to determine the times at which there are 8000 cells.

a. Solve for t by factoring.

b. What method did you use to factor?

c. Which values for t are reasonable and which are unreasonable? Explain.

d. Graph the function for $0 \le t \le 20$ using your calculator.

SOLUTION:

a. Substitute 8000 for $P(t)$ and solve for x.

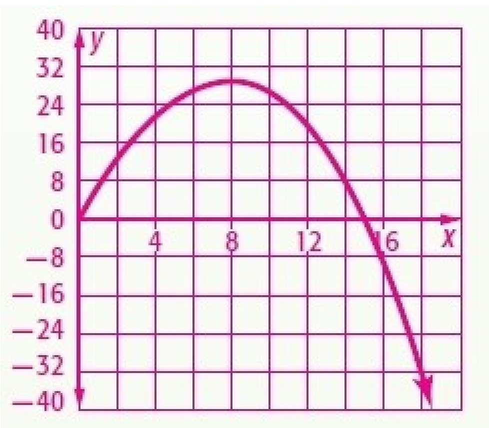
$$\begin{aligned} -0.012t^3 - 0.24t^2 + 6.3t + 8000 &= 8000 \\ 12t^3 + 240t^2 - 6300t &= 0 \\ 12t\left(t^2 + 20t - 525\right) &= 0 \\ 12t(t+35)(t-15) &= 0 \end{aligned}$$

Therefore, $t = 0$, 15 and –35.

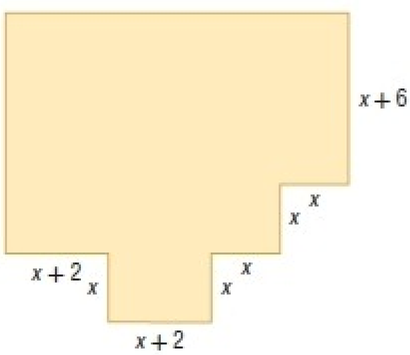
b. Sample answer: Subtract 8000 from both sides. Then convert the decimals to integers and factor out 120t, then factor the remaining trinomial.

c. 15 and 0 are reasonable, and –35 is unreasonable because time cannot be negative.

d.



73. **HOME BUILDING** Alicia's parents want their basement home theater designed according to the diagram.

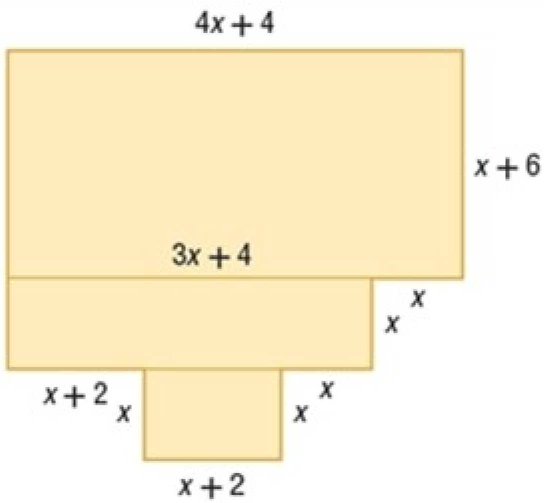


a. Write a function in terms of x for the area of the basement.

b. If the basement is to be 1366 square feet, what is x?

SOLUTION:

a.



The area of the basement in terms of x is $(4x+4)(x+6)+(3x+4)x+x(x+2)$.

$$(4x+4)(x+6)+(3x+4)x+x(x+2)$$
$$=4x^2+28x+24+3x^2+4x+x^2+2x$$
$$=8x^2+34x+24$$

Therefore, $f(x) = 8x^2 + 34x + 24$.

b. Substitute 1366 for $f(x)$ and solve for x.

$$8x^2+34x+24=1366$$
$$8x^2+34x-1342=0$$
$$4x^2+17x-671=0$$

$$x=\frac{-17\pm\sqrt{17^2-4(4)(-671)}}{2(4)}$$
$$=\frac{-17\pm\sqrt{11025}}{8}$$
$$=\frac{-17\pm105}{8}$$

Therefore, x = 11 or –15.25.

–15.25 is irrelevant because length cannot be negative. Therefore, x = 11 ft.

74. **BIOLOGY** A population of parasites in an experiment can be modeled by $f(t) = t^3 + 5t^2 - 4t - 20$, where t is the time in days.

a. Use factoring by grouping to determine the values of t for which $f(t) = 0$.

b. At what times does the population reach zero?

c. Are any of the values of t unreasonable? Explain.

SOLUTION:

a. Substitute 0 for x and solve for x.

$$t^3 + 5t^2 - 4t - 20 = 0$$
$$(t-2)(t^2 + 7t + 10) = 0$$
$$(t-2)(t+2)(t+5) = 0$$

Therefore, the values of t are 2, –2 and –5.

b. The population reaches zero in 2, –2 and –5 days.

c. –2 and –5 are unreasonable because time cannot be negative.

Factor completely. If the polynomial is not factorable, write prime.

75. $x^6 - 4x^4 - 8x^4 + 32x^2 + 16x^2 - 64$

SOLUTION:

$$x^6 - 4x^4 - 8x^4 + 32x^2 + 16x^2 - 64 = x^6 - 12x^4 + 48x^2 - 64$$

Let $y = x^2$.

$$\begin{aligned} x^6 - 12x^4 + 48x^2 - 64 &= y^3 - 12y^2 + 48y - 64 \\ &= (y-4)(y^2 - 8y + 16) \\ &= (y-4)(y-4)(y-4) \\ &= (y-4)^3 \\ &= (x^2 - 4)^3 \\ &= (x^2 - 2^2)^3 \\ &= ((x-2)(x+2))^3 \\ &= (x-2)^3(x+2)^3 \end{aligned}$$

76. $y^9 - y^6 - 2y^6 + 2y^3 + y^3 - 1$

SOLUTION:

$$y^9 - y^6 - 2y^6 + 2y^3 + y^3 - 1 = y^9 - 3y^6 + 3y^3 - 1$$

Let $x = y^3$.

$$\begin{aligned} y^9 - 3y^6 + 3y^3 - 1 &= x^3 - 3x^2 + 3x - 1 \\ &= (x-1)(x^2 - 2x + 1) \\ &= (x-1)(x-1)(x-1) \\ &= (x-1)^3 \\ &= (y^3 - 1)^3 \\ &= (y^3 - 1^3)^3 \\ &= ((y-1)(y^2 + y + 1))^3 \\ &= (y-1)^3 (y^2 + y + 1)^3 \end{aligned}$$

77. $x^6 - 3x^4y^2 + 3x^2y^4 - y^6$

SOLUTION:

$$\begin{aligned} x^6 - 3x^4y^2 + 3x^2y^4 - y^6 &= (x^2)^3 - 3(x^2)^2 y^2 + 3x^2 (y^2)^2 - (y^2)^3 \\ &= (x^2 - y^2)^3 \\ &= [(x+y)(x-y)]^3 \\ &= (x+y)^3 (x-y)^3 \end{aligned}$$

78. **CCSS SENSE-MAKING** Fredo's corral, an enclosure for livestock, is currently 32 feet by 40 feet. He wants to enlarge the area to 4.5 times its current area by increasing the length and width by the same amount.

a. Draw a diagram to represent the situation.

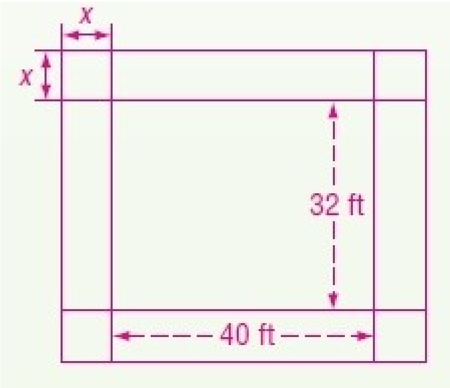
b. Write a polynomial equation for the area of the new corral. Then solve the equation by factoring.

c. Graph the function.

d. Which solution is irrelevant? Explain.

SOLUTION:

a.



b. The area of the corral is 32 × 40 = 1280 ft^2.
The area of the new corral in terms of *x* is $(32+2x)(40+2x)$.
The area of the new corral is 1280 ×4.5 = 5760 ft^2.

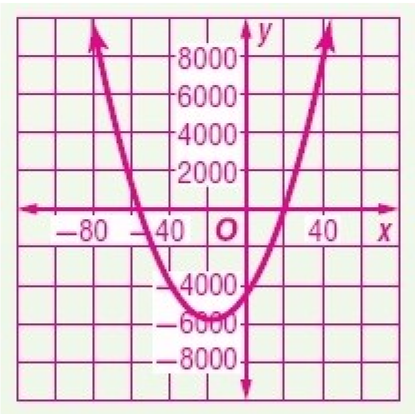
$$(32+2x)(40+2x)=5760$$
$$4x^2+144x+1280=5760$$

Solve for *x*.

$$4x^2+144x-4480=0$$
$$x^2+36x-1120=0$$
$$(x+56)(x-20)=0$$

Therefore, *x* = –56 or 20.

c.



d. –56 is irrelevant because length cannot be negative.

5-5 Solving Polynomial Equations

79. **CHALLENGE** Factor $36x^{2n} + 12x^n + 1$.

SOLUTION:

$36x^{2n} + 12x^n + 1$

Let $y = x^n$.

$$\begin{aligned} 36y^2 + 12y + 1 &= 36y^2 + 6y + 6y + 1 \\ &= 6y(6y+1) + 1(6y+1) \\ &= (6y+1)(6y+1) \\ &= (6y+1)^2 \\ &= (6x^n+1)^2 \end{aligned}$$

80. **CHALLENGE** Solve $6x - 11\sqrt{3x} + 12 = 0$.

SOLUTION:

$6x - 11\sqrt{3x} + 12 = 0$

Let $y = \sqrt{3x}$.

$$\begin{aligned} 2y^2 - 11y + 12 &= 0 \\ 2y^2 - 8y - 3y + 12 &= 0 \\ 2y(y-4) - 3(y-4) &= 0 \\ (2y-3)(y-4) &= 0 \end{aligned}$$

By Zero Product Property:

$y - 4 = 0$	or	$2y - 3 = 0$
$y = 4$	or	$y = \frac{3}{2}$
$\sqrt{3x} = 4$	or	$\sqrt{3x} = \frac{3}{2}$
$3x = 16$	or	$3x = \frac{9}{4}$
$x = \frac{16}{3}$	or	$x = \frac{3}{4}$

The solutions are $\frac{16}{3}, \frac{3}{4}$.

81. **REASONING** Find a counterexample to the statement $a^2 + b^2 = (a + b)^2$.

SOLUTION:

Sample answer: $a = 1$, $b = -1$

82. **OPEN ENDED** The cubic form of an equation is $ax^3 + bx^2 + cx + d = 0$. Write an equation with degree 6 that can be written in *cubic* form

SOLUTION:

Sample answer: A polynomial with degree 6 that can be written in cubic form will have a first term in the form $(x^2)^3$. Then the second term will be in the form of $(x^2)^2$.

$12x^6 + 6x^4 + 8x^2 + 4 = 12(x^2)^3 + 6(x^2)^2 + 8(x^2) + 4$

83. **WRITING IN MATH** Explain how the graph of a polynomial function can help you factor the polynomial.

SOLUTION:

Sample answer: The factors can be determined by the x-intercepts of the graph. An x-intercept of 5 represents a factor of $(x – 5)$.

84. **SHORT RESPONSE** Tiles numbered from 1 to 6 are placed in a bag and are drawn to determine which of six tasks will be assigned to six people. What is the probability that the tiles numbered 5 and 6 are the last two drawn?

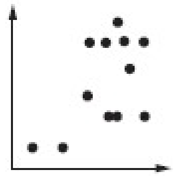
SOLUTION:

$$\begin{aligned} P(5 \text{ and } 6) &= \frac{1}{6}\cdot\frac{1}{5}+\frac{1}{6}\cdot\frac{1}{5} \\ &= \frac{1}{30}+\frac{1}{30} \\ &= \frac{2}{30} \\ &= \frac{1}{15} \end{aligned}$$

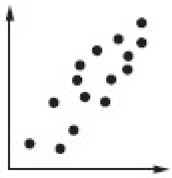
5-5 Solving Polynomial Equations

85. **STATISTICS** Which of the following represents a negative correlation?

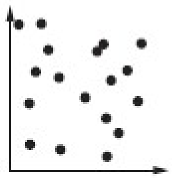
A.



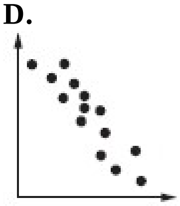
B.



C.



D.



SOLUTION:

Option A and C have no correlation.
Option B has positive correlation.
Option D has negative correlation.
Therefore, option D is the correct answer.

86. Which of the following most accurately describes the translation of the graph $y = (x + 4)^2 - 3$ to the graph of $y = (x - 1)^2 + 3$?

F down 1 and to the right 3
G down 6 and to the left 5
H up 1 and to the left 3
J up 6 and to the right 5

SOLUTION:

The graph $y = (x + 4)^2 - 3$ is translated 6 units up and 5 units to the right and positioned at $y = (x - 1)^2 + 3$.
Therefore, option J is the correct answer.

87. **SAT/ACT** The positive difference between k and $\frac{1}{12}$ is the same as the positive difference between $\frac{1}{3}$ and $\frac{1}{5}$. Which of the following is the value of k?

A. $\frac{1}{60}$
B. $\frac{1}{20}$
C. $\frac{1}{15}$
D. $\frac{13}{60}$
E. $\frac{37}{60}$

SOLUTION:

$$\begin{aligned}\left|k-\frac{1}{12}\right| &= \left|\frac{1}{3}-\frac{1}{5}\right| \\ &= \left|\frac{2}{15}\right| \\ &= \frac{2}{15} \\ k &= \frac{2}{15}+\frac{1}{12} \\ &= \frac{39}{180} \\ &= \frac{13}{60}\end{aligned}$$

Therefore, option D is the correct answer.

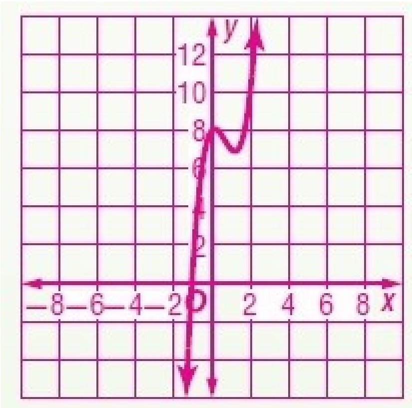
5-5 Solving Polynomial Equations

Graph each polynomial function. Estimate the *x*-coordinates at which the relative maxima and relative minima occur.

88. $f(x) = 2x^3 - 4x^2 + x + 8$

SOLUTION:

Graph the polynomial $f(x) = 2x^3 - 4x^2 + x + 8$.

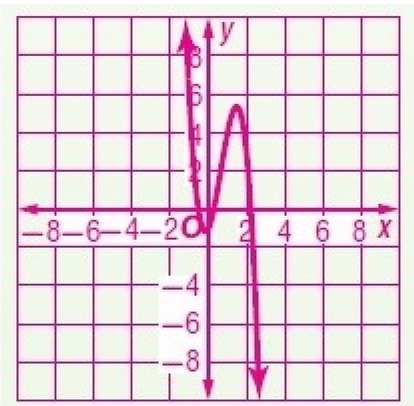


The x-coordinate at the relative maximum at $x \approx 0.1$.
The x-coordinate at the relative minimum at $x \approx 1.2$

89. $f(x) = -3x^3 + 6x^2 + 2x - 1$

SOLUTION:

Graph the polynomial $f(x) = -3x^3 + 6x^2 + 2x - 1$.



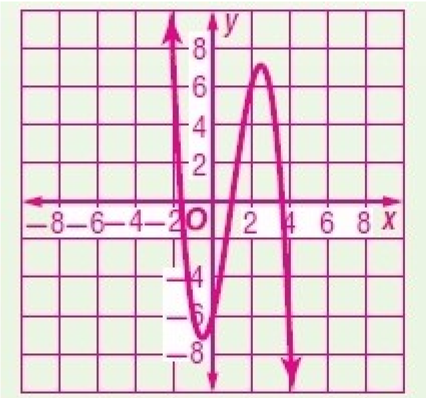
The x-coordinate at the relative maximum at $x \approx 1.5$.
The x-coordinate at the relative minimum at $x \approx 0.1$

5-5 Solving Polynomial Equations

90. $f(x) = -x^3 + 3x^2 + 4x - 6$

SOLUTION:

Graph the polynomial $f(x) = -x^3 + 3x^2 + 4x - 6$.



The x-coordinate at the relative maximum at $x \approx 2.5$.
The x-coordinate at the relative minimum at $x \approx -0.5$

State the degree and leading coefficient of each polynomial in one variable. If it is not a polynomial in one variable, explain why.

91. $f(x) = 4x^3 - 6x^2 + 5x^4 - 8x$

SOLUTION:

The degree of the polynomial is 4.
The leading coefficient of the polynomial is 5.

92. $f(x) = -2x^5 + 5x^4 + 3x^2 + 9$

SOLUTION:

The degree of the polynomial is 5.
The leading coefficient of the polynomial is –2.

93. $f(x) = -x^4 - 3x^3 + 2x^6 - x^7$

SOLUTION:

The degree of the polynomial is 7.
The leading coefficient of the polynomial is –1.

94. **ELECTRICITY** The impedance in one part of a series circuit is $3 + 4j$ ohms, and the impedance in another part of the circuit is $2 - 6j$. Add these complex numbers to find the total impedance of the circuit.

SOLUTION:

$$\begin{aligned}(3+4j)+(2-6j) &= (3+2)+(4-6)j \\ &= 5-2j\end{aligned}$$

95. **SKIING** All 28 members of a ski club went on a trip. The club paid a total of $478 for the equipment. How many skis and snowboards did they rent?

SOLUTION:
Let x and y be number of skis and snowboards respectively.
The system of equations represent this situation is:

$$\begin{aligned} x + y &= 28 \\ 16x + 19y &= 478 \end{aligned}$$

The solution of the system of equations is (18, 10).
Therefore, they rented 18 skins and 10 snowboards.

96. **GEOMETRY** The sides of an angle are parts of two lines whose equations are $2y + 3x = -7$ and $3y - 2x = 9$. The angle's vertex is the point where the two sides meet. Find the coordinates of the vertex of the angle.

SOLUTION:
Solve the system of equations $2y + 3x = -7$ and $3y - 2x = 9$.
The solution is (–3, 1).
Therefore, the coordinates of the vertex of the angle is (–3, 1).

Divide.

97. $(x^2 + 6x - 2) \div (x + 4)$

SOLUTION:

$$\begin{array}{r} x + 2 \\ x + 4 \overline{\big) x^2 + 6x - 2} \\ \underline{(-)\ x^2 + 4x} \\ 2x - 2 \\ \underline{(-)\ 2x + 8} \\ -10 \end{array}$$

$$\left(x^2 + 6x - 2\right) \div \left(x + 4\right) = x + 2 - \frac{10}{x + 4}$$

5-5 Solving Polynomial Equations

98. $(2x^2 + 8x - 10) \div (2x + 1)$

SOLUTION:

$$\begin{array}{r l}
 & x + 3.5 \\
2x+1 \big) & 2x^2 + 8x - 10 \\
 & \underline{(-)\ 2x^2 + x} \\
 & \quad 7x - 10 \\
 & \quad \underline{(-)\ 7x + 3.5} \\
 & \quad\quad -13.5
\end{array}$$

$$\left(2x^2 + 8x - 10\right) \div \left(2x + 1\right) = x + 3.5 - \frac{13.5}{2x + 1}$$

99. $(8x^3 + 4x^2 + 6) \div (x + 2)$

SOLUTION:

$$\begin{array}{r l}
 & 8x^2 - 12x + 24 \\
x+2 \big) & 8x^3 + 4x^2 + 6 \\
 & \underline{(-)\ 8x^3 + 16x^2} \\
 & \quad -12x^2 \\
 & \quad \underline{(-) -12x^2 - 24x} \\
 & \quad\quad 24x + 6 \\
 & \quad\quad \underline{(-)\ 24x + 48} \\
 & \quad\quad\quad -42
\end{array}$$

$$\left(8x^3 + 4x^2 + 6\right) \div \left(x + 2\right) = 8x^2 - 12x + 24 - \frac{42}{x + 2}$$